THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2055 Tutorial 2 (Sep 21)

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1. For a set S of numbers, a member c of S is called the maximum of S provided that it is an upper bound for S. Prove that a set S of numbers has a maximum if and only if it is bounded above and $\sup S$ belong to S. Give an example of a set S of numbers that is nonempty and bounded above but has no maximum.

2. Let $S = \{1 - (-1)^n/n : n \in \mathbb{N}\}$. Find $\inf S$ and $\sup S$.

3. Suppose that A is a nonempty set of real numbers that is both bounded above and bounded below, and that $\inf A = \sup A$. Prove that the set A consists of exactly one number.

4. Show that if A and B are bounded above in $\mathbb{R}$, then $A \cup B$ is a bounded above. Show that $\sup(A \cup B) = \sup\{\sup A, \sup B\}$.

5. (Optional)For a function $f(x)$, we can define another function

$$f^*(p) = \sup_{x \in \mathbb{R}} \{px - f(x)\}$$

If $g = e^x$, calculate g^* and g^{**}.